INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICS
HIGHER LEVEL
PAPER 1

Monday 7 May 2001 (afternoon)

2 hours

Name

Number

INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9400, Texas Instruments TI-85.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. Let $f(t) = t^{\frac{1}{3}}\left(1 - \frac{1}{2t^{\frac{5}{3}}}\right)$. Find $\int f(t)\,\mathrm{d}t$.

Working:

Answer:

2. Solve $2\sin x = \tan x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Working:

Answers:

3. Give a full geometric description of the transformation represented by the matrix $\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$.

Working:

Answer:

4. Find the gradient of the tangent to the curve $3x^2 + 4y^2 = 7$ at the point where $x = 1$ and $y > 0$.

Working:

Answer: _____

5. Let $f : x \mapsto \sqrt{\frac{1}{x^2} - 2}$. Find

(a) the set of real values of x for which f is real and finite;

(b) the range of f.

Working:

Answers:

(a) _____

(b) _____

6. A machine produces packets of sugar. The weights in grams of thirty packets chosen at random are shown below.

Weight (g)	29.6	29.7	29.8	29.9	30.0	30.1	30.2	30.3
Frequency	2	3	4	5	7	5	3	1

Find unbiased estimates of

(a) the mean of the population from which this sample is taken;

(b) the variance of the population from which this sample is taken.

Working:

Answers:

(a) _____

(b) _____

7. The nth term, u_n, of a geometric sequence is given by $u_n = 3(4)^{n+1}$, $n \in \mathbb{Z}^+$.

(a) Find the common ratio r.

(b) Hence, or otherwise, find S_n, the sum of the first n terms of this sequence.

Working:

Answers:

(a) _____

(b) _____

8. Let $f : x \mapsto \dfrac{\sin x}{x}$, $\pi \leq x \leq 3\pi$. Find the area enclosed by the graph of f and the x-axis.

Working:

Answer:

_____

9. Find the equation of the line of intersection of the two planes $-4x + y + z = -2$ and $3x - y + 2z = -1$.

Working:

Answer:

10. $(z + 2i)$ is a factor of $2z^3 - 3z^2 + 8z - 12$. Find the other two factors.

Working:

Answers:

11. Given that $P(X) = \frac{2}{3}$, $P(Y \mid X) = \frac{2}{5}$ and $P(Y \mid X') = \frac{1}{4}$, find

(a) $P(Y')$;

(b) $P(X' \cup Y')$.

Working:

Answers:

(a) _____

(b) _____

12. Find an equation of the plane containing the two lines

$$x - 1 = \frac{1 - y}{2} = z - 2 \text{ and } \frac{x + 1}{3} = \frac{2 - y}{3} = \frac{z + 2}{5}.$$

Working:

Answer:

_____

13. Z is the standardised normal random variable with mean 0 and variance 1. Find the value of a such that $P(|Z| \leq a) = 0.75$.

Working:

Answer:

14. Given that $z = (b + \mathrm{i})^2$, where b is real and positive, find the **exact** value of b when $\arg z = 60°$.

Working:

Answer:

15. X is a binomial random variable, where the number of trials is 5 and the probability of success of each trial is p. **Find the values of p if $P(X=4)=0.12$.**

Working:

Answer:

16. Find the general solution of the differential equation $\frac{dx}{dt} = kx(5 - x)$, where $0 < x < 5$, and k is a constant.

Working:

Answer:

17. An astronaut on the moon throws a ball vertically upwards. The height, s metres, of the ball, after t seconds, is given by the equation $s = 40t + 0.5at^2$, where a is a constant. If the ball reaches its maximum height when $t = 25$, find the value of a.

Working:

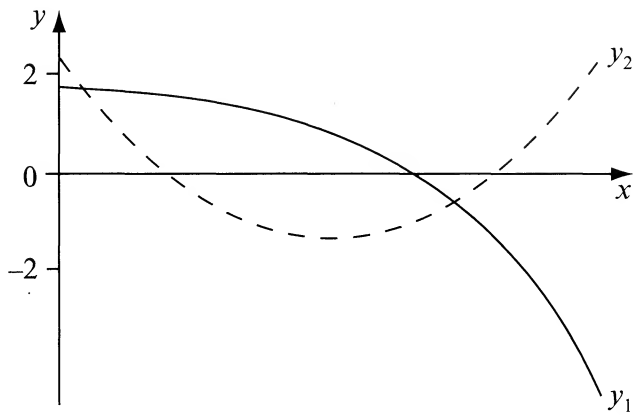
Answer:

18. The equation $kx^2 - 3x + (k + 2) = 0$ has two distinct real roots. Find the set of possible values of k.

Working:

Answers:

19. The diagram shows the graph of the functions y_1 and y_2.



On the same axes sketch the graph of $\frac{y_1}{y_2}$. Indicate clearly where the x-intercepts and asymptotes occur.

Working:

20. The function f is given by $f: x \mapsto e^{(1+\sin \pi x)}$, $x \geq 0$.

(a) Find $f'(x)$.

Let x_n be the value of x where the $(n+1)^{th}$ maximum or minimum point occurs, $n \in \mathbb{N}$. (*i.e.* x_0 is the value of x where the first maximum or minimum occurs, x_1 is the value of x where the second maximum or minimum occurs, *etc*).

(b) Find x_n in terms of n.

Working:

Answers:

(a) _____

(b) _____